Geometric Design of Highways

Vertical Alignments

Chapter 3
9/17/08 and 9/22/08
Principles of Highway Alignment

- The alignment of a highway is a three-dimensional problem measured in x, y, and z dimensions.

Figure 3.1 Highway alignment in three dimensions.
Three dimensional problem reduced to two:

1. Horizontal alignment of road shown in x and z coordinates called the plan view.

2. Vertical alignment of the road is shown on the y axis and is called the profile view.
Principles of Highway Alignment

Figure 3.2 Highway alignment in two-dimensional views.
Principles of Highway Alignment

• Further simplified by using highway position and length instead of x and y.

• Distances are measured in terms of stations, with each station consisting of 100ft of highway alignment distance.

• For example, a point 4250 ft from a specified origin is said to be at station 42+50

• The point of origin is at station 0 + 00
Vertical Alignment

• Specifies the elevations of points along a roadway.
• Elevations are determined by need to provide proper drainage and driver safety.
• A primary concern of vertical alignment is to establish a transition between two roadway grades by means of a vertical curve.
Vertical Alignment

- Two types of Vertical Curves:
  1. Crest Vertical Curves.
  2. Sag Vertical Curves.
Vertical Curve Properties

- The initial road grade is called $G_1$ the final road grade is called $G_2$ and is typically given in percent.
- PVC is the point of the vertical curve.
- The point of intersection of the initial tangent grade and the final tangent grade is the point of vertical intersection (PVI).
- The absolute value of the difference between $G_1$ and $G_2$ is called $A$ and is given in percent.
- The point of intersection of the vertical curve with the final tangent grade is called the PVT.
- The length ($L$) of the vertical curve is the horizontal distance between PVC and PVT.
- Equal Tangent, if PVC to PVI is $L/2$. 
Vertical Curve Design

- Vertical Curves maximum grades depend on:
  - Design Speed
  - Type of Terrain
  - Type of Highway
  - Length of Grade

- Grades also affect:
  - fuel consumption
  - speed
  - accidents (speed differential)
Vertical Curve Formula:

\[ y = ax^2 + bx + c \]

assuming a constant rate of change of slope and equal tangent lengths.

When \( x = 0 \), \( y = C \) = elevation on curve

\[ \frac{dy}{dx} = 2ax + b = \text{slope (rise / run)} \]

at \( x = 0 \) \( \Rightarrow \) slope is \( \frac{dy}{dx} = b = G_1 \) (the initial slope)
Vertical Curve Formula:

\[ \frac{d^2y}{dx^2} = 2a \]

= rate of change of slope

\[ 2a = \frac{(G_2 - G_1)}{L} \]

\[ a = \frac{(G_2 - G_1)}{2L} \]

therefore:

\[ y_x = \left(\frac{(G_2 - G_1)}{2L}\right) x^2 + G_1 x + \text{Elevation at PVC} \]

Where: \( G_2, G_1(\%) \), \( L \text{ (sta.)} \), \( x \text{ (sta.)} \)
High / Low Point on Curve

If not PVC or PVT,
(G1 and G2 have different signs)

\[
dy/dx = 2ax + b = 0
\]

\[
b = G_1
\]

\[
a = (G_2 - G_1) / 2L
\]

then:

\[
X = -G_1L / (G_2 - G_1)
\]
To find Elevation at the “turning” point

Substitute “x” in:

\[ y_x = \frac{(G_2 - G_1)}{2L} x^2 + G_1 x + \text{Elevation of PVC} \]
Example 3.1:

A 600ft m equal tangent sag vertical curve has the PVC at station 170 +00 and elevation 1000ft. The initial grade is -3.5 % and the final grade is 0.5 %.

Determine the elevation and stationing of the PVI, PVT, and the lowest point on the curve.
Example 3.1:

Solution: Determining Low point on Curve

If the initial and final grades are opposite in sign, the low point on the curve will occur when the first derivative of the parabolic function is zero.

\[
\frac{dx}{dy} = 2ax + b = 0
\]

When the initial and final grades are not opposite in sign, the low point on the curve will be at the PVC or PVT.

This problem is Case 1.
Curve-through-a-point

• Sometimes curve must be designed so that the elevation of a specific location is met.
• Example: 1. to have a roadway connect with another, or 2. to have roadway at some specified elevation to pass under another roadway
Example 3.2

An equal tangent vertical curve is to be constructed between grades of $-2.0\%$ (initial) and $1.0\%$ (final). The PVI is at station 110 + 00 and at elevation 420 ft. Due to a street crossing the roadway, the elevation of the roadway at station 112 + 00 must be at 424.5 ft. Design the curve.
Properties of Vertical Curves

- Offsets from the initial tangent (profile grade line) are very important in vertical curve design and construction. Y is defined as the offset at any distance, x, from the PVC.
Other Vertical Curve Relationships

\[ Y = \frac{A}{200L} x^2 \]

\[ Y_m = \frac{AL}{800} \]

\( Y \) = the offset at any distance, \( x \), from the PVC
\( Y_m \) = the midcurve offset
\( A \) = the absolute value of the difference in the grades (\( | G_1 - G_2 | \)) (%)
\( L \) = the length of the vertical curve
\( x \) = the distance from the PVC
Vertical Alignment

• In above equations, 200 is used in the denominator instead of, 2, because $A (lG_1 - G_2 l)$, is expressed in percent rather than ft/ft.
Vertical Alignment Cont.

- By definition, $K$ (rate of vertical curvature) is the horizontal distance in ft (meters) required to affect a 1% change in the slope of a vertical curve. It has several uses including simplifying the computation of the high/low points of vertical curves. $K$ is calculated as follows.

\[
K = \frac{L}{A} \quad \text{(eq 3.10)}
\]

- $L$ is in ft (meters), $A$ is in percent.
Vertical Alignment

- The following equation is used to determine the horizontal distance $x_{hl}$ to the high/low point in meters, given that the point does not occur at the PVC or PVT

$$x_{hl} = K \times |G_1| \quad \text{(eq. 3.11)}$$

- $G_1$ is the absolute value of the initial grade in percent. Watch the units. In equation (3.3) $G_1$ was in m/m.
Example 3.3

• A curve has initial and final grades of +3% and -4%, respectively, and is 700ft long. The PVC is at elevation 100ft. Graph the vertical curve elevations and the slope of the curve against the length of the curve. Compute K value and use it to locate the high point of the curve (distance from the PVC).
Example 3.4

• A vertical curve crosses a 4ft diameter pipe at right angles. The pipe is located at station 110 + 85 and its centerline is at elevation 1091.60 ft. The PVI of the vertical curve is at station 110 + 00 and elevation 1098.40 m. The vertical curve is equal tangent, 600ft long, and connects an initial grade of +1.20% and a final grade of −1.08%. Using offsets, determine the depth, below the surface of the curve, to the top of the pipe, and determine the station of the highest point on the curve.
Geometric Design of Highways

Vertical Curve Sight Distance (cont.)

Chapter 3
Minimum Stopping-Sight Distance

• Construction of a vertical curve is generally a costly operation requiring the movement of significant amounts of earthen material.

• The challenge is to minimize construction costs, usually by making the vertical curve as short as possible, while still providing an adequate level of safety.

• The safety level is adequate if motorists have sufficient sight distance to safely stop if there is an obstruction in the roadway.
Stopping-Sight Distance

- The necessary stopping sight distance (SSD) is the summation of the vehicle stopping-sight distance (Eq. 2.48) and the distance traveled during perception/reaction time (Eq. 2.50).

\[
SSD = \frac{v_1^2}{2g(\alpha \pm G)} + v_1 \times t_r \quad \text{(Eq. 3.12)}
\]

- \(V_1\) is the initial speed in \(\text{ft/s}\), \(g\) is \(32.2 \ \text{ft/s}^2\), \(a = 11.2 \ \text{ft/s}^2\), \(G\) is the grade, and \(t_r\) is the perception/reaction time = 2.5 seconds.
Design Speed and Minimal Curve Length

• The design speed of the highway is defined as the maximum safe speed at which a highway can be negotiated assuming near worst case conditions (wet-weather conditions)

• It is common practice to set minimum curve lengths ranging from 100 to 325ft. A common alternative is to set the minimum curve length at 3 times the design speed (with mph).
## Stopping-Sight Distance

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<th>Design speed (mi/h)</th>
<th>U.S. Customary</th>
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*Note: Brake reaction distance is based on a time of 2.5 s; a deceleration rate of 11.2 ft/s² (3.4 m/s²) is used to determine calculated stopping sight distance.*

“Crest” Curve Design:

Figure 3.5 Stopping-sight-distance considerations for crest vertical curves.
Stopping-Sight Distance and Crest Vertical Curve Design

• Length of curve (L) is the critical concern in providing sufficient SSD on vertical curves.
• Longer curves provide more SSD, but are more costly to construct.
• What is needed is an expression for minimum curve lengths given a required SSD.
• Crest and sag vertical curves are considered separately in developing such an expression.
“Crest” Curve Design:
Sight Distance **less** than length of vertical curve: \((S<L)\)

\[
L = \frac{AS^2}{200\left(\sqrt{H_1} + \sqrt{H_2}\right)^2}
\]

- \(L\) = length of vertical curve (ft)
- \(S\) = Sight Distance (ft)
- \(A\) = Algebraic difference in grades (%)
- \(H_1\) = Height of eye above roadway surface (ft)
- \(H_2\) = Height of object above roadway surface (ft)
"Crest" Curve Design:
Sight Distance greater than length of vertical curve: \((S > L)\)

\[
L = 2S - \frac{200(\sqrt{H_1} + \sqrt{H_2})^2}{A}
\]

- \(L\) = length of vertical curve (ft)
- \(S\) = Sight Distance (ft)
- \(A\) = Algebraic difference in grades (%)
- \(H_1\) = Height of eye above roadway surface (ft)
- \(H_2\) = Height of object above roadway surface (ft)
Stopping-Sight Distance for Crest Vertical Curve Design

• The assumption $L > SSD$ is almost always made in practice for two reasons:
  
  • The assumption that $L > SSD$ is usually a safe one. When it is not, the use of the $L > SSD$ formula gives longer curve lengths thus the error is on the conservative side giving a longer curve length than is needed.

• At low values of $A$, it is possible to compute negative minimum curve lengths.
“Crest” Curve Design:

When you assume AASHTO guidelines:

- \( H_1 = 3.5 \text{ft} \) (driver eye height)
- \( H_2 = 2.0 \text{ft} \) (object height)
- \( S = \text{SSD} \)

\[
L = \frac{AS^2}{200\left(\sqrt{H_1} + \sqrt{H_2}\right)^2}
\]

\[
K = \frac{SSD^2}{2158}
\]

\[
L = \frac{A \times SSD^2}{2158}
\]

- \( L \) = length of vertical curve (ft)
- \( SSD \) = Stopping-Sight Distance (ft)
- \( A \) = Algebraic difference in grades (%)
### Rate of Vertical Curve K

<table>
<thead>
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<th>U.S. Customary</th>
<th>Metric</th>
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\( \* \)Rate of vertical curvature, \( K \), is the length of curve per percent algebraic difference in intersecting grades (\( A \)): 
\[ K = \frac{L}{A}. \]

Example 3.5

• A highway being designed to AASHTO guidelines with a 70mph design speed, and at one section, an equal tangent vertical curve must be designed to connect grades of +1.0% and -2.0%.

Determine the minimum length of curve necessary to meet SSD requirements.
Stopping-Sight Distance for Sag Vertical Curve Design

• Sag vertical curve design differs from crest vertical curve design because it is governed by nighttime conditions. In daylight, sight distance on a vertical curve is unrestricted.

• The critical concern for sag vertical curve design is the length of roadway illuminated by the vehicles' headlights.
Stopping-Sight Distance for Sag Vertical Curve Design
Stopping-Sight Distance and Sag Vertical Curve Design

- The minimum length of a sag vertical curve is a function of the height of the headlights above the road, and the angle of the headlight beam relative to the plane of the car.

\[
For \ S < \ L \quad L = \frac{A \cdot S^2}{200(H + S \tan \beta)}
\]

\[
For \ S < \ L \quad L = \frac{A \cdot SSD^2}{400 + 3.5 \cdot SSD}
\]

\[
H = 2.0\text{ft} \quad \beta = 1^\circ
\]
Stopping-Sight Distance for Sag Vertical Curve Design

As with crest vertical curves, K-values can be computed by assuming \( L > SSD \), thus for sag vertical curves (with \( L_m = KA \))

\[
K = \frac{SSD^2}{400 + 3.5 \times SSD}
\]
K Value for Sag Vertical Curve Design

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<th>Design speed (mi/h)</th>
<th>Stopping sight distance (ft)</th>
<th>Rate of vertical curvature, $K^*$</th>
<th>Design speed (km/h)</th>
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*Rate of vertical curvature, $K^*$, is the length of curve per percent algebraic difference in intersecting grades $(A)$: $K = L/A$. 
Example 3.8

- An existing tunnel needs to be connected to a newly constructed bridge with sag and crest vertical curves. The profile view of the tunnel and bridge is shown in Fig 3.8. Develop a vertical alignment to connect the tunnel and bridge by determining the highest possible common design speed for the sag and crest (equal tangent) vertical curves needed. Compute the stationing and elevations of PVC, PVI, and PVT curve points.
Example 3.8

Figure 3.8 Profile view (vertical alignment diagram) for Example 3.8.
Example 3.9

- Consider the conditions described in Ex. 3.8. Suppose a design speed of only 35 mph is needed. Determine the lengths of curves required to connect the bridge while keeping the connecting grade as small as possible.
Example 3.9

Tunnel floor elevation 100 ft
Station 0 + 00, $PVC_s$

Station 12 + 00, $PVT_c$

Bridge deck elevation 140 ft

$G_{con}$
Example 3.10

- Two sections of highway are separated by 1800ft as shown in Fig. 3.10. Determine the curve lengths required for a 60mph vertical alignment to connect these two highway segments while keeping the connecting grade as small as possible.
Example 3.10
Passing-Sight Distance and Crest Vertical Curve Design

• In two-lane highway design it is sometimes desirable to provide adequate passing sight distance (PSD) on a vertical curve.

• PSD is only an issue on crest vertical curves. The view is unobstructed on sag curves, and at night motorists can see the headlights of oncoming vehicles.
Passing Sight Distance

CE 3201 Introduction to Transportation Engineering
Passing-Sight Distance and Crest Vertical Curve Design

- Passing sight distance for design consists of four distances.
  1. Initial maneuver distance, which includes perception/reaction time and time to accelerate to speed needed to pass.
  2. Distance traversed while in the left lane.
  3. Clearance length about vehicle being passed.
  4. Distance traversed by opposing vehicle during the maneuver.
Passing-Sight Distance and Crest Vertical Curve Design

• Determination of these distances takes into account vehicle acceleration, and speeds of passing, passed, and opposing vehicles.

• The summation of these four distances gives a single required PSD.
Passing-Sight Distance and Crest Vertical Curve Design

- $H_1$ remains 3.5 ft, but $H_2$ increases to 3.5, the assumed height of an opposing vehicle.

For $PSD < L$:

$$L = \frac{AS^2}{200(\sqrt{H_1} + \sqrt{H_2})^2}$$

$\rightarrow$

$$L = \frac{A \cdot PSD^2}{2800}$$

$$K = \frac{PSD^2}{2800}$$
Passing-Sight Distance and Crest Vertical Curve Design

• Minimum PSDs and K values are computed in table 3.4.

• Notice that K values are much higher than those computed for SSD. As a result designing a crest curve for PSD is expensive due to the length of curve required and not very common.
Passing-Sight Distance and Crest Vertical Curve Design

Table 3.4: Design Controls for Crest Vertical Curves Based on Passing Sight Distance

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<th>Design speed (mi/h)</th>
<th>Passing sight distance (ft)</th>
<th>Rate of vertical curvature, ( K^* )</th>
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*Rate of vertical curvature, \( K^* \), is the length of curve per percent algebraic difference in intersecting grades (A): \( K = L/A \).

Example 3.10

- An equal-tangent crest vertical curve is 4000ft long and connects a +2.5% and –1.5% grade. If the design speed of the roadway is 55mph, does this curve have adequate passing-sight distance?